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Radiation and kinetic properties of quasiplane multidimensional domain walls

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Abstract. In the framework of the variational approach to the collective-variable formalism the radiation field generated by the multidimensional domain walls is found and the dissipation of these walls is considered. Also, the drift velocity of the domain wall in an external field is calculated.

1. Introduction

The aim of this article is to determine basic low-energy properties of quasiplane domain walls using the variational approach to the method of collective variables. This concerns first of all the determination of radiation generated by the developing domain wall and the wall damping connected with this radiation as well as the determination of low-temperature kinetic properties of the wall in a multidimensional situation.

The problem of a microscopic description of nonlinear systems in terms of collective variables is reduced to two important issues. The first consists of the necessity of adequate separation of initial dynamic variables into ones that are coherent with respect to collective variables and incoherent (radiation) components, and the second is connected with the determination of a natural functional link between initial dynamic variables of the system and collective variables. These questions may be answered in the framework of the variational approach to the method of collective variables [1].

For the Klein–Gordon nonlinear system with the Hamiltonian

$$H=\tfrac{1}{2}\int \mathrm{d}^3x\,(\Pi^2(x)+(\nabla\Phi(x))^2+2U(\Phi(x))) \tag{1}$$

the link between the initial variables $\Phi(x)$. $\Pi(x)$ and the collective variables X_α, P_α is determined by formulae

$$\Phi(x)=\Phi_c(x,X)+\sum_\lambda a_\lambda\Psi_\lambda(x,X) \qquad \pi(x)\equiv(1-\wp)\Pi(x)=\sum_\lambda b_\lambda\Psi_\lambda(x,X)$$
$$\Pi(x)=\pi(x)+(P-G)Q^{-1}\delta\Phi_c/\delta X \tag{2}$$

and based on using the constraints

$$\langle(\delta\Phi_c/\delta X_\alpha)(\Phi-\Phi_c)\rangle=0$$

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where

$$G_\alpha = \langle \pi \delta\Phi/\delta X_\alpha \rangle \qquad Q_{\alpha\beta} = \langle (\delta\Phi/\delta X_\alpha)(\delta\Phi_c/\delta X_\beta) \rangle \qquad \langle A \rangle \equiv \int d^3x A(x).$$

$\Psi_\lambda/(x, X)$ are the basis vectors in the subspace orthogonal to the projection operator $\wp$, the core of which has the form

$$\begin{aligned} \wp(x, x') &= (\delta\Phi_c(x)/\delta X) M^{(0)^{-1}} (\delta/_c(x')/\delta X) \\ M^{(0)}_{\alpha\beta} &= \langle (\delta\Phi_c/\delta X_\alpha)(\delta\Phi_c/\delta X_\beta) \rangle. \end{aligned} \tag{3}$$

The rest of the field degrees of freedom are described by the coefficients a_λ, b_λ, which, like variables X_α, P_α, are canonically conjugated, $\{a_\lambda, b_{\lambda'}\} = \delta_{\lambda\lambda'}$, $\{X_\alpha, P_\beta\} = \delta_{\alpha\beta}$. The quasi-static ansatz $\Phi_c(x, X)$ appearing in these formulae is determined from the variational principle and satisfies the equation

$$(1 - \wp)\delta H/\delta\Phi_c(x) = 0.$$

This equation transforms into the equation for stationary states $\delta H/\delta\Phi(x) = 0$ if the collective coordinates X_α are the degeneration parameters of vacuum solutions. The results adduced are the generalization of the canonical transformation [2] for the case of arbitrary collective variables.

The transition to new variables itself does not provide for sufficiency of the method because the collective flux is determined not only by the variables X_α, P_α but also by the coherent components $\bar{a}_\lambda(X, P)$, $\bar{b}_\lambda(X, P)$ of the coefficients a_λ, b_λ. These coherent components are also determined from the variational principle, and the corresponding equations have the form

$$(1 - \wp)(\delta H/\delta\Phi(x) + PM^{-1}\delta\Pi(x)/\delta X)(\Phi = \bar{\Phi}, \pi = \bar{\pi}) = 0 \tag{4}$$

$$\bar{\pi}(x) = (1 - \wp)\bar{\Pi}(x) \qquad \bar{\Pi}(x) = PM^{-1}(\delta\Phi(x)/\delta X)(\Phi = \bar{\Phi}) \tag{5}$$

where $M_{\alpha\beta} = \langle (\delta\Phi/\delta X_\alpha)(\delta\Phi/\delta X_\beta) \rangle (\Phi = \bar{\Phi})$.

The Hamiltonian of the system in new variables is written in the form

$$H = H_0(X, P) + H_2(X, P; \tilde{\Phi}, \tilde{\pi}) + V(X, P; \tilde{\Phi})$$

where

$$H_0(X, P) = H(\Phi = \bar{\Phi}, \Pi = \bar{\Pi}) = (\tfrac{1}{2})PM^{-1}P + H(\bar{\Phi}, 0) \tag{6}$$

is the Hamiltonian of collective variables.

$$\begin{aligned} &H_2(X, P; \tilde{\Phi}, \tilde{\pi}) = (\tfrac{1}{2})(\langle \tilde{\Phi} L(\Phi_c) \tilde{\Phi} \rangle + \langle \tilde{\pi}^2 \rangle - 2PM^{(0)^{-1}} \langle \tilde{\pi} \delta\tilde{\Phi}/\delta X \rangle) + O(P^2) \\ &L(\Phi_c) = -\Delta + U''(\Phi_c) \end{aligned}$$

is the part of the Hamiltonian that is bilinear in the incoherent components $\tilde{\Phi} = \Phi - \bar{\Phi}$, $\tilde{\pi} = \pi - \bar{\pi}$ written in the linear approximation over collective momenta. The expression for $V(X, P; \tilde{\Phi})$ has the form

$$V(X, P; \tilde{\Phi}) = \sum_{n=3}^{\infty} \frac{1}{n!} \langle U^{(n)}(\bar{\Phi})\tilde{\Phi}^n \rangle.$$

It should be stressed that fluctuations $\tilde{\Phi}$, $\tilde{\pi}$ are not canonical variables, but for small P_α their Poisson brackets are close to Poisson brackets for canonical variables,

$$\{\tilde{\Phi}(x), \tilde{\Phi}(x')\} \sim O(P^3) \qquad \{\tilde{\pi}(x), \tilde{\pi}(x')\} \sim O(P^5)$$
$$\{\tilde{\Phi}(x), \tilde{\pi}(x')\} = (1 - \wp)(x, x') + O(P^4).$$

2. The dynamic ansatz $\bar{\Phi}(x; X, P)$ and the membrane Hamiltonian

The formulae of the preceding section are of general form and they are not connected with the particular choice of collective variables. If a particular problem is dealt with, the choice of collective variables is defined by physical considerations. Obviously, the multidimensional domain wall may be regarded as the spatial membrane, for which $X_\alpha(\sigma)$ will play the role of collective coordinates if the internal structure of the wall is not taken into account. If, for example, the double-degenerated ground state is realized at $\Phi(x) = \pm\Phi_0$, then this membrane may be identified with the surface at which the field $\Phi(x)$ changes its sign. Evidently the description of domain walls with the help of a surface assumes that the characteristic curvature radii of this surface exceed the domain wall thickness considerably. Under these conditions one may reproduce the obvious solution of equation (4) in the form of the following dynamic ansatz:

$$\bar{\Phi}(x; X, P) = u_c\left(\frac{h(\sigma(x))}{m_0\sqrt{g}}\, z(x)\right). \tag{7}$$

Here $u_c(z)$ is the one-dimensional soliton of the nonlinear Klein–Gordon equation, $m_0 = \int dz\, u_c'^2(z)$ is its mass, $h(\sigma) = \sqrt{P^2(\sigma) + m_0^2 g(\sigma)}$, $z(x) = [\boldsymbol{x} - \boldsymbol{X}(\sigma(x))]\boldsymbol{n}(\sigma(x))$, σ^i are the coordinates on the surface, $\sigma^i(x)$ is the point on the surface nearest to x, $\boldsymbol{n}(\sigma)$ is the normal to the surface at the point σ, $g = \det g_{ik}$, $g_{ik} = \boldsymbol{X}_{,i}\boldsymbol{X}_{,k}$ is the metric tensor on the surface, $P_\alpha(\sigma)$ is the membrane momentum at the point σ canonically conjugated to the collective coordinate $X_\alpha(\sigma)$, $X_{,i} \equiv \partial X/\partial\sigma^i$.

Let us now define the Hamiltonian of the collective variables corresponding to solution (7) for the dynamical ansatz $\bar{\Phi}(x; X, P)$. Using the substitution for variables mentioned above we find

$$\delta\Phi_c(x)/\delta X_\alpha(\sigma) = -\Delta(\sigma(x), \sigma)n_\alpha(\sigma(x))u_c'(z(x))$$

where $\Delta(\sigma, \sigma')$ is the δ-function on the surface. Taking into account that in the topical area near the surface of membrane the volume element has the form $dx \approx \sqrt{g}\, dz\, d^2\sigma$, we obtain for the matrix $M_{\alpha\beta}(\sigma, \sigma')$ (5)

$$M_{\alpha\beta}(\sigma, \sigma') = m(\sigma)n_\alpha(\sigma)n_\beta(\sigma)\sqrt{g(\sigma)}\Delta(\sigma, \sigma')$$

where

$$m(\sigma) = \int_{-\infty}^{+\infty} dz \left(\frac{\partial\bar{\Phi}(x; X, P)}{\partial z}\right)^2 = \frac{h(\sigma)}{\sqrt{g(\sigma)}}.$$

Due to the invariance of the theory under the arbitrary changes of the parameters $\sigma \to \sigma(\sigma')$ defining the coordinates on the membrane surface, the following constraints take place [3]:

$$P(\sigma)X_{,i}(\sigma)=0.$$

Namely, the momentum $P(\sigma)$ is orthogonal to the surface $X(\sigma)$ at the point σ and hence directed along the vector $\boldsymbol{n}(\sigma)$. Therefore, substituting the expression for $M_{\alpha\beta}(\sigma, \sigma')$ and formula (7) into (6) and using the identity $U(u_c)=(\frac{1}{2})(u_c')^2$ we obtain, after integrating over z, the Hamiltonian

$$H_0(X, P)=\int d^2\sigma\,(h(\sigma)+\lambda^i P(\sigma)X_{,i}(\sigma)) \tag{8}$$

describing the relativistic membrane in the non-covariant approach. In accordance with the Dirac approach [4], we added to this expression the term connected with the availability of the reparametrization constraints.

In the absence of folds, which is valid for small oscillations of the membrane, one can introduce the natural gauge condition for the components $X_\perp(\sigma)$ orthogonal to the total momentum of the system $P_{\rm tot}=\int d^2\sigma\, P(\sigma)$, putting $X_{\perp i}(\sigma)=\sigma^i$. In this case $g_{ij}=\delta_{ij}+X_{3,i}X_{3,j}$ and $\det g\cong 1+X_{3,i}^2$. Due to the coupling conditions $PX_{,i}=0$, the transverse momenta $P_{\perp j}(\sigma)=-P_3(\sigma)X_{3,j}(\sigma)$ will be the second-order quantities in gradients and momenta of the longitudinal displacements $X_3(\sigma)$. In the leading approximation over this parameters, the Hamiltonian (8) will have the form

$$H_0=(\tfrac{1}{2})\int d^2\sigma\,(m_0X_{3,i}^2(\sigma)+m_0^{-1}P_3^2(\sigma)). \tag{9}$$

This is the Hamiltonian of small oscillations of a membrane. In terms of the amplitudes of these oscillations a, a^* determined by expansions

$$\begin{aligned} X_3(\sigma)&=X_c+\sum_{k\neq 0}(2m_0kS)^{-1/2}(a_k+a_{-k}^*)\exp(-\mathrm{i}\boldsymbol{k\sigma})\\ P_3(\sigma)&=(R_{\rm tot}/S)-\mathrm{i}\sum_{k\neq 0}(m_0k/2S)^{1/2}(a_k-a_{-k}^*)\exp(-\mathrm{i}\boldsymbol{k\sigma}) \end{aligned} \tag{10}$$

the Hamiltonian H_0 has the form

$$H_0=(P_{\rm tot}^2/2m_0S)+\sum_{k\neq 0}ka_k^*a_k=(P_{\rm tot}^2/2m_0S)+\sum_{k\neq 0}kn_k$$

where $S=\int d^2\sigma$ is the area of the domain wall, $X_c=\int d^2\sigma\, X_3(\sigma)$ is the coordinate of the plane associated with the domain wall, and n_k, ϑ_k are the action-angle variables defined by formulae $a_k=\sqrt{n_k}\exp(-\mathrm{i}\vartheta_k)$, $a_k^*=\sqrt{n_k}\exp(-\mathrm{i}\vartheta_k)$.

3. Radiation of spatial modes

The oscillations of the domain walls generate fluctuations of the field variables $\tilde{\Phi}$, $\tilde{\pi}$ and, consequently, lead to emission of spatial modes. For a small curvature of the membrane, the fluctuations $\tilde{\Phi}$, $\tilde{\pi}$ will also be small. Under this condition we find the equations for radiation components, calculating the coefficients of the zero and first powers of the field fluctuations $\tilde{\Phi}$, $\tilde{\pi}$ in the leading approximation over gradients and momenta of displacements. The zeroth over the fluctuations terms in the equations of motion occurs when calculating the Poisson brackets $\{\tilde{\Phi}, H_0\}$, $\{\tilde{\pi}, H_0\}$ with the subsequent substitution $a \to \bar{a}$, $b \to \bar{b}$, and represents the external forces generating these fluctuations. Directly from formulae (5), (7) and (9) it may be proved that the contribution of the Poisson brackets $\{\tilde{\pi}, H_0\}$ to the equations of motions is small compared with the contribution of the bracket

$$\{\tilde{\Phi}(x), H_0\}(a=\bar{a}, b=\bar{b}) = -(1/m_0)z(x)u_c'(z(x))P_3(\sigma(x))\Delta X_3(\sigma(x)) \equiv \pi_c(x). \tag{11}$$

What determines the terms uniform in fluctuations, in the leading approximation, is the fragment of the Hamiltonian H_2:

$$\{\tilde{\Phi}(x), H_2\} \cong \tilde{\pi}(x) \qquad \{\tilde{\pi}(x), H_2\} \cong -L\tilde{\Phi}(x). \tag{12}$$

The difference of the potential $U''(\Phi_c)$ in the operator L from $\Omega^2 \equiv U''(\Phi_0)$ is not significant in this problem. Therefore, from equations (11) and (12) one finds

$$\ddot{\tilde{\Phi}}(x) + L_0\tilde{\Phi}(x) = \dot{\pi}_c(x) \qquad L_0 = -\Delta + \Omega^2. \tag{13}$$

Using the expression for the retarding Green function

$$G_{\rm ret}(t) = \theta(t)\sin(t\sqrt{L_0})/\sqrt{L_0}$$

one may write the solution of equation (13) in the form

$$\begin{aligned}\tilde{\Phi}(x, t) &= \int_{-\infty}^{t} \mathrm{d}t'\, G_{\rm ret}(t-t')\dot{\pi}(x, t) \\ &= -\frac{\mathrm{i}\Lambda}{(\Lambda - \mathrm{i}\eta)^2 - L_0}\frac{1}{(2\pi)^3}\int \mathrm{d}^3q\, \mathrm{e}^{\mathrm{i}qx}\pi_{cq}(t) \qquad \eta \to +0\end{aligned} \tag{14}$$

where $\Lambda = -\mathrm{i}\sum k\partial/\partial\vartheta_k$ is the Liouville operator of surface modes. It is convenient to calculate the integral determining the Fourier component π_{cq} in the variables $\boldsymbol{\sigma}$, z that are related to the coordinates $\boldsymbol{x}$ by the equation $\boldsymbol{x} = \boldsymbol{\sigma} + \boldsymbol{X}_3(\sigma) + \boldsymbol{n}(\sigma)z$. Then using the long-wavelength approximation $q_3X_3(\sigma) \ll 1$ as well as weak displacement gradients one finds that

$$\pi_{cq} = \int \mathrm{d}^3x\, \pi_c(x)\exp(-\mathrm{i}qx) \cong 2\mathrm{i}\Phi_0 m_0^{-1}\overline{z^2}\int \mathrm{d}^2\sigma\, \exp(-\mathrm{i}q\sigma)q_3P_3(\sigma)\Delta_\sigma X_3(\sigma) \tag{15}$$

where $\overline{z^2} \equiv (2\Phi_0)^{-1}\int_{-\infty}^{+\infty} \mathrm{d}z\, u_c'(z)z^2$ is the factor determining the domain wall thickness.

Substituting equations (10) and (15) into (14) and calculating the integral over σ and q in the asymptotic region $x_3 \to \pm\infty$, we find

$$\begin{aligned}\tilde{\Phi}(x, t) = \mp\frac{\Phi_0\overline{z^2}}{4m_0S}\sum_{k,k'}\sqrt{kk'}(k+k')^2\exp\{-\mathrm{i}[(\boldsymbol{k}+\boldsymbol{k}')\boldsymbol{x}_\perp \\ + (k+k')t \mp p(\boldsymbol{k}, \boldsymbol{k}')x_3]\}\theta(p^2)a_ka_{k'} + \text{c.c.} \\ p(\boldsymbol{k}, \boldsymbol{k}') = \sqrt{2(kk' - \boldsymbol{k}\boldsymbol{k}') - \Omega^2}.\end{aligned}$$

This expression determines the radiation field in the form of waves diverging from the wall. The condition $p^2(\boldsymbol{k}, \boldsymbol{k}') > 0$ defines the region in the wave vector space of surface excitations, the binary collisions of which lead to this radiation.

4. Relaxation of the quasiplane domain wall

The emission of spatial modes results in the damping of the domain wall oscillations. This damping is described by the equations for the action variables n_k. In the leading approximation over gradients and $\tilde{\Phi}$, one has

$$\dot{n}_k = -\partial H/\partial \vartheta_k \cong \int d^2\sigma\, (\partial P_3(\sigma)/\partial \vartheta_k)\langle(\delta\bar{\Phi}/\delta P_3(\sigma))L_0\tilde{\Phi}\rangle.$$

Using the solution for $\tilde{\Phi}$ (equations (14) and (11)) and noting that

$$\delta\bar{\Phi}(x)/\delta P_3(\sigma) \cong m_0^{-1} z(x) P_3(\sigma(x)) u_c'(z(x))\Delta(\sigma, \sigma(x))$$

where $\Delta(\sigma, \sigma')$ is the δ-function on the surface, one finds

$$\dot{n}_k = -[(2\pi)^3 m_0^2]^{-1} \int d^3q \left| \int dz\, z u_c'(z) \exp(iq_3 z) \right|^2 \int d^2\sigma\, d^2\sigma' \exp[i\boldsymbol{q}_\perp(\boldsymbol{\sigma} - \boldsymbol{\sigma}')]$$
$$\times \frac{\delta P_3(\sigma)}{\delta \vartheta_k} P_3(\sigma) \frac{i(q^2+\Omega^2)\Lambda}{q^2+\Omega^2-(\Lambda - i\eta)^2} P_3(\sigma')\Delta_{\sigma'} X_3(\sigma'). \tag{16}$$

It is well known that rapid randomization of the phases occurs in the system of interacting oscillators. Therefore, one may simplify further analysis if one substitutes expansions (4) into equation (16) and then average the right-hand side of this equation over the surface mode phases ϑ_k. At $S \to \infty$ such averaging reduces to the establishment of binary couplings $[a_1 a_2^*] = \delta_{1,2} n_1$. Then, making the sequential integration over $\boldsymbol{\sigma}$, $\boldsymbol{\sigma}'$, $\boldsymbol{q}_\perp$, q_3 and over z, z', one obtains

$$\dot{n}_k = -\int d^2k'\, J(\boldsymbol{k}, \boldsymbol{k}') n_k n_k'$$

$$J(\boldsymbol{k}, \boldsymbol{k}') = \frac{1}{m_0}\left(\frac{\Phi_0}{2\pi}\overline{z^2}\right)^2 kk'(k+k')^4 p(\boldsymbol{k}, \boldsymbol{k}')\theta[p^2(\boldsymbol{k}, \boldsymbol{k}')].$$

This nonlinear integral equation determines the dissipation of surface modes due to their binary inelastic collisions. There are a few exact singular solutions of this equation. Some of them have the form

$$n_k(t) = N[2\pi k_0(1 + N\bar{J}(k_0, k_0)t)]^{-1}\delta(k - k_0) \qquad \bar{J}(k_1, k_2) \equiv (2\pi)^{-1}\int_0^{2\pi} d\varphi\, J(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

$$n_k(t) = N[1 - \exp(-J(\boldsymbol{k}_1, \boldsymbol{k}_2)Nt)]^{-1}[\delta(\boldsymbol{k} - \boldsymbol{k}_1) + \exp(-J(\boldsymbol{k}_1, \boldsymbol{k}_2)Nt)\delta(\boldsymbol{k} - \boldsymbol{k}_2)].$$

5. Low-temperature drift of the multidimensional domain wall

The non-equilibrium properties of solitons in one-dimensional theories have been investigated in many papers [5, 6]. The variational approach to the collective-variable formalism permits one to analyse a multidimensional situation. As an example, consider the evolution of the system under the action of the external perturbation removing the degeneration of the ground state. In this case one of the phases becomes unstable and its volume begins to shrink. This process may be presented locally as the induced motion of the quasiplane domain boundary in the direction of the unstable phase. According to the linear response theory the average velocity of this motion is determined by the expression

$$\bar{\dot{X}}=T^{-1}\int_0^\infty \mathrm{d}t\,\langle\langle\{H, V\}\dot{X}_c(t)\rangle\rangle \tag{17}$$

where X_c is the domain wall coordinate (10), $V=\int \mathrm{d}^3\,xv(\Phi(x))$, $v(\Phi)=-v(-\Phi)$ is the external perturbation, and angular brackets $\langle\langle\ \rangle\rangle$ denote averaging over the equilibrium Gibb's ensemble corresponding to the Hamiltonian (9).

Obviously, the main low-temperature contribution is determined by terms containing the minimal power of dynamic variables. Guided by this consideration in analysing the expression

$$\{H, V\}=-\langle\Pi v'(\Phi)\rangle=-\langle(\pi+\dot{X}QM^{-1}\delta\Phi_c/\delta X)v'(\Phi)\rangle$$

and noting that

$$\delta\Phi_c(x)/\delta X_\alpha(\sigma)=-\Delta(\sigma(x),\sigma)n_\alpha(\sigma(x))u_c'(z(x)) \tag{18}$$

we find that only one term in equation (17) is singular at $t\to\infty$ and is proportional to the correlator $\langle\langle\dot{X}_c\dot{X}_c(t)\rangle\rangle$. Thus, calculating the spatial integral one gets

$$\bar{\dot{X}}=2v(\Phi_0)ST^{-1}D. \tag{19}$$

where $D=\int_0^\infty \mathrm{d}t\langle\langle\dot{X}_c\dot{X}_c(t)\rangle\rangle$ is the diffusion coefficient.

In the memory function formalism [7] one may obtain for the diffusion coefficient the following presentation:

$$D=\langle\langle\dot{X}_c^2\rangle\rangle^2\hat{F}^{-1}(0) \qquad \hat{F}(0)=\int_0^\infty \mathrm{d}t\,\langle\langle\ddot{X}_c\exp(iQ\Lambda Qt)\ddot{X}_c\rangle\rangle \tag{20}$$

where $Q=1-\dot{X}_c\rangle\rangle\langle\langle\dot{X}_c^2\rangle\rangle^{-1}\langle\langle\dot{X}_c$ is the projection operator. In the leading low-temperature approximation the operator Q should be substituted by unity and for Λ one should take the Liouville operator corresponding to the Hamiltonian of non-interacting elementary excitations

$$H_0=P_{\mathrm{tot}}^2/2m_0S+\sum ka_k^*a_k+(\tfrac{1}{2})\langle\tilde{\Phi}L(u_c)\tilde{\Phi}+\tilde{\pi}^2\rangle.$$

The first two terms describe the motion of the domain wall as a whole and the excitations of its surface modes, the third one defines the fluctuations of the spatial (incoherent) mode on the background of the plane domain wall.

The simple analysis accounting for the energy δ-function presence in equation (20) shows that the fragment of the acceleration determined by the expression

$$\ddot{X}_c=\{(\partial H/\partial P_{\mathrm{tot}}), H\}=\frac{1}{3!m_0S}(\langle\tilde{\Phi}'\tilde{\pi}\rangle\langle U'''(u_c)\tilde{\Phi}^3\rangle)$$

$$=-\frac{1}{2m_0S}\langle U'''(u_c)\tilde{\Phi}^2(1-\wp)\tilde{\Phi}'\rangle \tag{21}$$

makes the main contribution to equation (20).

Using perturbation to calculate the thermodynamic average one should pay attention to the peculiarities inherent in the multidimensional problem. Remember that due to expansions (2) the incoherent components $\tilde{\Phi}$, $\tilde{\pi}$ belong to the eigensubspace of the projection operator $1-\wp$. According to equations (3) and (15) in the limiting case of the resting plane of the domain wall, the core of the operator $\wp$ assumes the form

$$\wp(x,x')=m_0^{-1}u_c'(z(x))u_c'(z(x'))\Delta(\sigma(x),\sigma(x'))\equiv\wp_3(z(x),z(x'))\Delta(\sigma(x),\sigma(x')).$$

This shows that not only the zeroth translational mode of the operator $L(u_c)$ is excluded from the spectrum of fluctuations $\tilde{\Phi}(x)$ but also the whole bunch of modes associated with this mode and transverse to the normal. If one bears in mind that in contrast to the one-dimensional model the spectrum of the operator $L(u_c)$ possesses no gap, this means that the condition $\wp\tilde{\Phi}(x)=0$ restores this gap in the spectrum of incoherent components. This is important for constructing low-temperature expansions. It prevents the appearance of principal-value integrals with dangerous denominators in the perturbation theory. Naturally, the degrees of freedom excluded from field variables due to the condition $\wp\tilde{\Phi}(x)=0$ manifest themselves as collective variables describing the domain wall.

Thus, expanding the fluctuations $\tilde{\Phi}$, $\tilde{\pi}$ over eigenfunctions $e^{-ik\sigma}\varphi_\lambda(z)$ of the operator $L(u_c)$ and substituting equation (21) into equation (20) one gets, after averaging over the Gibbs distribution,

$$D^{-1}=\frac{\pi T}{8S}\sum_{k,\lambda}\Delta(k_1+k_2+k_3)(\omega_{\lambda_1k_1}\omega_{\lambda_2k_2}\omega_{\lambda_3k_3})^{-2}$$
$$\times(|G_{\lambda_3\lambda_2,\lambda_1}|^2+2\,\mathrm{Re}\,G_{\lambda_1\lambda_2,\lambda_3}G_{\lambda_1\lambda_3,\lambda_2})$$
$$\times[\delta(\omega_{\lambda_1k_1}-\omega_{\lambda_2k_2}-\omega_{\lambda_3k_3})+2\delta(\omega_{\lambda_1k_1}+\omega_{\lambda_2k_2}-\omega_{\lambda_3k_3})] \tag{22}$$

where

$$G_{\lambda_1\lambda_2,\lambda_3}=\langle\varphi_{\lambda_1}\varphi_{\lambda_2}U'''(u_c)(1-\wp_3)\varphi'_{\lambda_3}\rangle$$

$\omega_{k\lambda}^2=k^2+\omega_\lambda^2$, and $\omega_\lambda\neq0$ is the spectrum of the one-dimensional operator $L(u_c)$ (note that the diffusion coefficient $D\sim S^{-1}$ and the drift velocity (19) remains finite at $S\to\infty$). The low-temperature behaviour of the diffusion coefficient $D\sim T^{-1}$ coincides with the low-temperature dependence for the one-dimensional model established in [6], and equation (22) modifies the expression for the diffusion coefficient of the one-dimensional model [8] by accounting for transverse phonons. Concerning the results obtained in this section, one needs to bear in mind that the expression for D (equation (22)) is valid at $T\ll\Omega$. It is a condition for the application of the low-temperature expansions. As for the drift velocity, which according to equations (19) and (22) is proportional to T^{-2}, it must be weak compared with the heat velocity of spatial phonons $v_T\sim\sqrt{T/\Omega}$. Obviously, if the external field $v(\Phi_0)$ is sufficiently weak, then a temperature interval exists where these conditions do not contradict each other.

6. Summary

This paper determines the properties of the quasiplane multidimensional domain walls of the nonlinear Klein–Gordon equation. The studies have been performed on the grounds of the variational approach to the collective-variable formalism. In the framework of this approach the fragments of the initial variables have been determined that are incoherent with respect to collective variables. In the asymptotic region these fragments constitute the radiation field generated by the subsystem of collective variables. Calculations of this radiation are easy and are not connected with specificity of the model considered. The paper stresses the important role of the constraints $\langle(\delta\Phi_c/\delta X_\alpha)(\Phi-\Phi_c)\rangle=0$ in forming the spectrum of the incoherent component, this constraints being the basis of expansion (2).

References

[1] Tsukanov V D 1992 *J. Phys. A: Math. Gen.* **25** 6099
[2] Tomboulis E 1975 *Phys. Rev.* D **12** 1678
[3] Collins P A and Tucker R W 1976 *Nucl. Phys.* B **112** 150
[4] Dirac P A M 1964 *Lectures on Quantum Mechanics* (New York: Yeshiva University)
[5] Wada Y and Schrieffer J R 1978 *Phys. Rev.* B **18** 3897
Collins M A, Blumen A, Currie J F and Ross J 1979 *Phys. Rev.* B **19** 3630
Sahni P S and Mazenko G F 1979 *Phys. Rev.* B **20** 4674
Marchesoni F 1987 *Phys. Rev.* A **36** 4559
[6] Kunz C 1987 *Phys. Rev.* A **34** 510
[7] Mori H 1965 *Prog. Theor. Phys. (Kyoto)* **34** 423
[8] Tsukanov V D 1990 *Teor. Mat. Fiz.* **84** 366